



Date: 03-11-2018
Time: 01:00-04:00

Dept. No.

Max. : 100 Marks

Section A

Answer ALL questions:

10 × 2 = 20

1. If $y = \sin(ax + b)$, find y_n .
2. Find the length of the subtangent for the parabola $y^2 = 4ax$.
3. If $A = \begin{pmatrix} 2 & 4 \\ 6 & 12 \end{pmatrix}$. Find the rank of A.
4. Sum to infinity of the series $1 + \frac{\log 2}{1!} + \frac{(\log 2)^2}{2!} + \dots$
5. Find the Laplace transform of $at^2 + bt + c$.
6. Find $L^{-1}\left(\frac{s}{s^2 - a^2}\right)$.
7. Write down the power series expansion for $\sin \theta$ and $\cos \theta$.
8. Prove that $\sin ix = i \sinh x$.
9. Two dice are thrown. What is the probability that the sum of the numbers is greater than 8?
10. If a Poisson variate X is such that $P(X = 1) = 2P(X = 2)$, find the mean.

Section B

Answer any FIVE questions:

5 × 8 = 40

11. Find the maximum value of $\frac{\log x}{x}$ for positive values of x .
12. Find the angle of intersection of the cardioids $r = a(1 + \cos \theta)$ and $r = b(1 - \cos \theta)$.
13. Find the sum of the series $1 - \frac{3}{1!}\left(\frac{1}{4}\right) + \frac{3.6}{2!}\left(\frac{1}{4}\right)^2 - \frac{3.6.9}{3!}\left(\frac{1}{4}\right)^3 + \dots \infty$.
14. Verify Cayley Hamilton theorem for the matrix $\begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$.
15. Find $L\{f(t)\}$ where $f(t) = \begin{cases} e^{-t} & \text{when } 0 < t < 4 \\ 0 & \text{when } t > 4 \end{cases}$.
16. Express $\cos^5 \theta$ in a series of cosines of multiples of θ .
17. If $\cos(x + iy) = \cos \theta + i \sin \theta$, then show that $\cos 2x + \cosh 2y = 2$.
18. A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which (i) neither car is used, (ii) the proportion of days on which some demand is refused.

Section C

Answer any TWO questions:

2 × 20 = 40

19. a) If $y = \sin(m\sin^{-1}x)$, prove that $(1 - x^2)y_2 - xy_1 + m^2y = 0$ and $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

(b) If a, b, c are three consecutive positive integers, show that

$$\log b = \frac{1}{2} \log a + \frac{1}{2} \log c + \frac{1}{2ac+1} + \frac{1}{(2ac+1)^2} + \dots \infty. \quad (15 + 5)$$

20.a) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{pmatrix}$

b) Evaluate $L(t^2e^{-3t})$. (15 + 5)

21. Solve the equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t$ given that $y = \frac{dy}{dt} = 0$ when $t = 0$.

22. a) Expand $\frac{\sin 6\theta}{\sin \theta}$ in powers of $\cos \theta$.

b) Calculate the mean and standard deviation for the following data giving the age distribution of 542 members.

Age (in years)	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
No. of members	3	61	132	153	140	51	2

(5 + 15)
